Nanotechnology: is it the exploitation of quantum effects?

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It has been said that nanotechnology is the exploitation of quantum effects. This is certainly true insofar as some nanotechnologies do exploit quantum effects; this paper attempts to find the boundaries through a review of current research and the problem of decoherence.

Introduction

When Feynman boldly asserted that there was plenty of room at the bottom, which has subsequently become almost ubiquitous as an opening to the subject of nanotechnology, he was looking from the perspective of quantum electrodynamics. However, since then nanotechnology as an industry and research subject has grown almost exponentially and now there are many definitions of nanotechnology, but all have in common the concept that at least one dimension of a nano-object is in the nanoscale, consensually taken to be of 1 to 100 nm; this means that a futuristic computer processor with a 1 cm × 1 cm graphene–silicon chip will be in the nanoscale because the effective transistors on a graphene sheet are only one carbon atom thick (which is actually thinner than 1 nm).

So, referring to our title, is nanotechnology also characterized by the onset of quantum mechanics? Taking an absurd reductionist view, all the physical sciences—physics (except nuclear physics), chemistry and biology—depend upon the arrangement and interactions of atoms and in particular the orbits of their outer electrons. In practice, however, beyond a certain scale it becomes highly inefficient to describe the world in terms of quantum mechanical interactions and classical physics, chemistry and biology are adequate for predicting behaviour. Hence, might a more pragmatic view be preferable? Nanotechnology and nanosized objects exhibit characteristics often very different from the classical predictions based on bulk materials. It is this threshold of change that is of interest, where everyday behaviour in the human scale is no longer true; for example, gold is yellow-coloured but a gold nanoparticle exhibits all the colours of the rainbow depending on the particle size. Ramsden and Freeman[1]

[1] J.J. Ramsden and J. Freeman, The nanoscale. *Nanotechnology Perceptions* **5** (2009) 3–25.

Nanotechnology Perceptions **8** (2012) 195–204
Received 26 November 2012

attempted to classify the bulk–nano threshold point in terms of the optical, physical and chemical properties of a number of materials and concluded, depending on the phenomenon of interest, that the threshold of change in most instances was indeed within the currently accepted nanoscale range.

In many cases, this change could be predicted classically simply because objects within the nanoscale have a surface area for their atoms and molecules to interact that is so much greater, in comparison with their volume, than that of bulk material (or even than that of micro-objects). One therefore has to ask whether quantum predictions are required; for instance, drug delivery using a lipid or other nanoparticle cage is considered a medical application of nanotechnology, but classical chemistry and biology are sufficient to explain and predict the benefit of such a technique.

Most people would associate quantum effects with the scale of electrons and nuclei, which actually puts them in the realm of subatomic particles and, hence, beyond the reach of the nanoscale as only the largest atoms reach 1 nm. Not so for light, which exhibits wave–particle duality even in the human scale (e.g., interference patterns, which any schoolchild can demonstrate as Young's fringes on the dining room table). Similarly, NMR (nuclear magnetic resonance), PET (positron emission tomography), and possibly STM (scanning tunnelling microscopy) are familiar, especially the first two because of their use in medical diagnosis, but curiously only the last one appears to be considered nanotechnology even though all utilize quantum effects.

Already it is possible to see that the question posited in the title has no simple answer; quantum effects are exploited at the nanoscale in some of our electronics; consider tunnelling diodes and transistors, magnetic memory reading using the giant magnetoresistance effect (electron spin alignment in the outer electrons of the substrate), quantum dots, quantum computing and potentially neuroscience.

The problem the question really addresses is, at what scale do quantum effects dominate? Schrödinger's cat has become the seminal example demonstrating that the superposition of states fundamental to all quantum mechanics may have no realization in the human scale. Current understanding would have the cat in both the dead and alive states only if it were cooled to near 0 K in order to prevent decoherence of its quantum states.

Where do quantum effects become important?

Much progress has been made in recent years on the realization of practical quantum effects. The electronics industry driven by the mobile phone market has continued to follow Moore's law, especially prominently regarding memory growth and computational parallelism. This has led to the ever-decreasing size of silicon substrate patterns constituting electronic components and present generation processing chips use surface depth effects of about 20 nm. Increasingly, graphene is being investigated, which, in theory, would use only one carbon atom thickness, or about 0.2 nm. Unfortunately, at this scale tunnelling and other quantum effects will play a role and make conventional circuit design more difficult.

A more obvious application of quantum effects is in the use of quantum dots. These blobs of semiconductor between 1 and 10 nm in diameter, which provide an electron potential well, are characterized by the bandgap energy difference between electrons in the highest valence band from those in the lowest conduction band. By stimulating a valence band electron it rises to the conduction band and then drops back releasing a photon; the photon frequency is characterized by the bandgap and, generally, the smaller the dot the smaller the wavelength of the emitted light. Stimulation of the quantum dot can be achieved either by an electromagnetic field or absorption of broad-spectrum light that causes the dot to fluoresce. Major usage at the moment is in the medical field as markers, where the dots are attached to a biomolecule of interest and fluoresce characteristically under an optical microscope. More futuristic uses are for the replacement of light-emitting diodes, compared with which they are more energy-efficient, and are likely to both replace existing lighting devices and be used in future display screens.

Critical to any computing engine is memory storage and for quantum computing this means storing qubits. Any object with a defined quantum state can be used but in practical terms this tends to be spin of outer electrons in a potential well such as a quantum dot, electron Cooper pair spins in a superconducting loop, or nuclear spin in a doped atom in a substrate. The problem is that energy from the environment can interact with the object and either corrupt or destroy its spin state. As a consequence, most proposed quantum memory devices operate near absolute zero where the energy of the environment is minimized. An example is the development of qubit storage and read-out attempting to use current fabrication technology.[2] The silicon device consists of implanted phosphorus donors coupled to a metal–oxide–semiconductor single-electron transistor—compatible with current microelectronic technology. A spin lifetime of ~6 seconds at a magnetic field of 1.5 tesla was achieved at a temperature of 300 mK.

Recent advances indicate considerable progress to a point at which room-temperature qubit memory becomes a real possibility.[3] The qubit consists of a single ^{13}C nuclear spin in the vicinity of a nitrogen vacancy (typical of point defects in diamond) within an isotopically pure diamond crystal. The long qubit memory time was achieved via a technique involving *dissipative decoupling* of the single nuclear spin from its local environment.

The practical exploitation of a quantum computer is also moving a step nearer with the application of circuit design techniques from the electronics industry. Here, circuit quantum electrodynamics (cQED)[4] allows spatially separated superconducting qubits to interact via a superconducting microwave cavity that acts as a "quantum bus", making possible two-qubit entanglement and the implementation of simple quantum algorithms. An indium arsenide nanowire double quantum dot made by electron depletion at two points in the nanowire is coupled to a superconducting cavity achieving a charge–cavity coupling rate of about 30 MHz.

In terms of application, quantum computing has led to no earth-shattering revelations and so far demonstrations have mainly concentrated on showing that it is possible by solving a few basic problems. Shor's algorithm to factor a prime number using entangled qubits is the most demonstrated and the next step will be to scale up a few qubit calculations to many,[5] much as four-bit early processors are now replaced by at least 64-bit ones. When this occurs, classical

[2] A. Morello et al., Single-shot readout of an electron spin in silicon. *Nature* **467** (2010) 687–691.

[3] P.C. Maurer et al., Room-temperature quantum bit memory exceeding one second. *Science* **336** (2012) 1283–1286.

[4] K.D. Petersson et al., Circuit quantum electrodynamics with a spin qubit. *Nature* **490** (2012) 380–383.

[5] E. Martín-López, Experimental realization of Shor's quantum factoring algorithm using qubit recycling. *Nature Photonics* **6** (2012) 773–776.

cryptology will be in trouble as encoding relies on large prime numbers[6] and with Shor's algorithm on a quantum computer it is conceivable that any code may be decrypted. This will have a big impact on the banking industry, which currently relies on public key encryption for many financial transactions. At this level of computation it is also likely that difficult many-body problems and complex systems such as the environment may be amenable to detailed mathematical modelling.

The subject of quantum cryptology is receiving much attention, especially the concept of entangled particles, which can be separated so that any measurement on one affects the other instantaneously. For encrypted communication this means that an eavesdropper may be immediately detected by the sender, possibly removing the need for encryption in the first place. Most research has concentrated on entangled photons where the separation distance, initially through fibre-optic cable, has been steadily increased and now stands at about 144 km.[7] In fact this was achieved through air using a laser pulse focused through a telescope and received by telescope.

For most people, however, quantum computers are still the stuff of science fiction but increasingly there are demonstrations of quantum effects on the human scale. Some recent demonstrations have been with mechanical oscillators, chemical reactions and, perhaps most dramatically, entanglement. For example, using conventional cryogenic refrigeration, a microwave frequency mechanical oscillator—a "quantum drum"—was cooled to its quantum ground state and coupled to a quantum bit, which was used to measure the quantum state of the resonator.[8]

Quantum effects are realized in the Penning ionization reaction of argon and molecular hydrogen with metastable helium, leading to a sharp absolute ionization rate increase in the energy range corresponding to a few degrees kelvin down to 10 mK.[9] This is a sign that a quantum phenomenon known as scattering resonances due to tunnelling is occurring in the reactions. At low energies, the wave-like behaviour of the particles dominates: those waves that were able to tunnel through the potential barrier interfered constructively with the reflected waves upon collision. This creates a standing wave that corresponds to particles trapped in orbits around one another. Such interference occurs at particular energies and is marked by a dramatic increase in reaction rates.

Of two entangled photons, one was sent to a standard photon detector, while the other was amplified using a machine that generated a shower of photons with the same polarization, thereby, in theory, generating a micro–macro entangled state.[10] The beam of light produced by

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6 "Public key" encryption consists of a product of two large primes used to encrypt a message, and a "secret key" consisting of the primes used and necessary to decrypt the message. You can make the public key public, and everyone can use it to encrypt messages to you, but only you know the prime factors and can decrypt the messages. Everyone else would have to factor the number, which takes too long to be practicable, given the current state of the art of number theory.

7 R. Ursin et al., Entanglement-based quantum communication over 144 km. *Nature Physics* **3** (2007) 481–486.

8 A.D.O'Connell et al., Quantum ground state and single-phonon control of a mechanical resonator. *Nature* **464** (2010) 697–703.

9 A.B. Henson et al., Observation of resonances in Penning ionization reactions at sub-Kelvin temperatures in merged beams. *Science* **338** (2012)234–238.

10 Z. Merali, Quantum effects brought to light. Published online 28 April 2011 (http://www.nature.com/news/2011/110428/full/news.2011.252.html).

the amplifier could appear in one of two positions, and the location of the beam reflected the polarization state of the photons in the field. A team sat in the dark for hours, marking the position of the light spot over repeated runs of the experiment, for the first time seeing the effects of quantum entanglement with the naked eye.

These examples all reflect the increasing interest in exploring the boundaries of quantum effects but perhaps the most controversial aspect concerns the human brain and whether or not it is a quantum computer. Penrose,[11] with others, has suggested that the neurons of the brain, which contain microtubules, constitute a quantum computer. The tubulin microtubules, some 25 nm in diameter, self-assemble and connect the cell nucleus to axons and dendrites. Since tubulins within the microtubules can exist in two shapes that may form, it is postulated, a superposition of states it is proposed that they form qubits. Initial criticism centred on decoherence—surely the warm, wet environment of the brain would destroy quantum states? It was proposed that consciousness and freedom of choice could be explained as a superstition of states, which then collapse to the choice made. This view is still controversial even though the boundaries of quantum effects are moving into the ambient environment and macroscopic scale.

The problem of decoherence

Quantum effects are expected to yield significant benefits if they can be exploited at room temperature and in bulk materials. Most people think of quantum effects at the atomic scale where supercooled particle accelerators operating in high vacuum cause matter to disintegrate into subatomic particles. Perhaps the most bizarre effect is the superposition of two particles in an entangled state where, if the particles are separated, the result of an action on one particle to fix its state immediately and necessarily affects the quantum state of the other. Moreover, this exchange of information occurs truly instantaneously and, hence, at greater than the speed of light. Exploitation of this phenomenon is likely to have a massive impact on our communications technology—and demonstrates the incompatibility of the quantum and relativity theories.

Exploitation of entangled superposition states is the enabling facility of quantum computing, whereby entangled qubits in both states, zero and one, provide massive computational process parallelism.

The problem with all particles is, however, that they react with their environment; for example, a ball placed in a warm room will become warmer. For the ball, the quantum states of its constituent atoms and molecules will be random and the absorption of energy from the warm room will not change its structure, only its temperature. However, for an object where the quantum state such as spin up or spin down of outer electrons is important for its use, the interaction with the environment will affect its quantum state and destroy its use as, say, a qubit.

Our natural environment is around 285 K (15 °C) and any object at this temperature will radiate heat according to the Stefan–Boltzmann law with energy over time t given by

$$E = A\,t\,\sigma\,T^4\,, \tag{1}$$

where σ is the Stefan–Boltzmann constant, A is the surface area of the object and T is the absolute

[11] R. Penrose, *Shadows of the Mind: A Search for the Missing Science of Consciousness*. Oxford: University Press (1994).

temperature. Assuming this radiated energy E dominates the interaction of the particle with an observer or detector, then it will affect the coherence of the particle's quantum state.

An object with quantum states of interest will interact with the environment and lose its initial quantum state after a decoherence time t given by the interaction energy E such that

$$E\,t \geqslant h/2\,, \tag{2}$$

where h is Planck's constant. (The appendix gives a simple derivation of this Heisenberg-like expression.) Combining these two equations gives

$$t \geqslant \sqrt{[h/(2\,A\,\sigma\,T^4)]}\,. \tag{3}$$

Using the values from Table 1 then gives rise to the graphs of Figure 1 showing decoherence time as a function of the radius of a spherical object. The sizes range from the size of a hydrogen atom to that of a small cat for both ambient temperature and 300 mK.

From Figure 2 it is apparent that only a few electron volts of energy will reduce decoherence time from microseconds to a few attoseconds, 10^{-18} s, and although it is now possible to measure time in attoseconds, attempting computation in this timescale is at present impracticable: a modern laptop computer typically makes a single calculation in a few hundred picoseconds, 10^{-12} s. The fact that practical decoherence times are only possible at temperatures near absolute zero means that a laptop quantum computer is some way into the future.

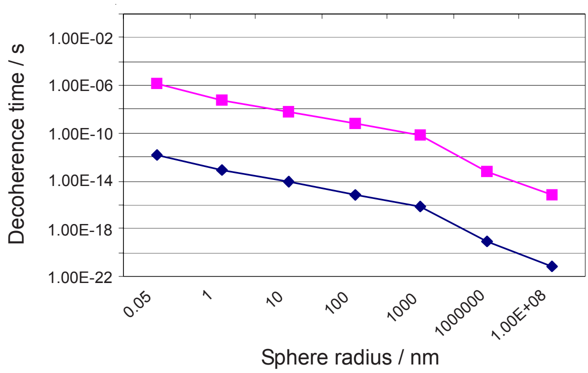

Figure 1. Decoherence time as a function of sphere radius. Diamonds: ambient temperature; squares: 0.3 K.

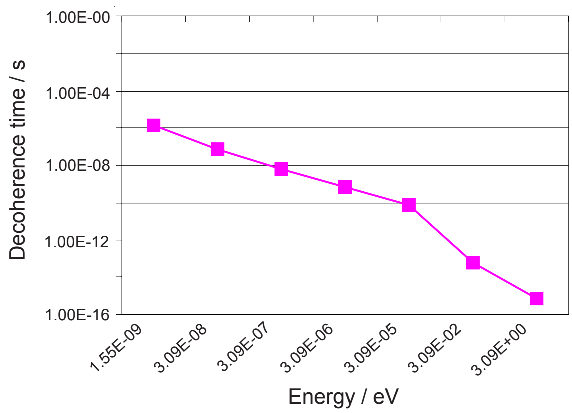

Figure 2. Decoherence time as a function of the energy radiated from a particle at 0.3 K.

Table 1. Values of various quantities, physical constants and functions.

Function	Symbol	Value	Units
Stefan-Boltzmann constant	σ	5.67×10^{-8}	$W\ m^{-2}\ K^{-1}$
Planck constant	h	6.626×10^{-34}	J s
Surface area	A	$4\ \pi\ r^2$	m^2
Electron charge	e	-1.602×10^{-19}	C
Electron rest mass	m_e	9.109×10^{-31}	kg
Electron energy	$m_e c^2$	0.5×10^6	eV
Joule	J	2.8×10^{-7}	kW h
Electron volt	eV	1.602×10^{-19}	J
Bohr radius	r_b	0.05	nm
Coulomb constant	k_e	8.987×10^9	$N\ m^2\ C^{-2}$
Coulomb potential at Bohr radius between 2 electrons	$k_e\, e^2 / r_b$	28	eV

In the case of the human brain it is difficult to see how it could be a quantum computer. From Figure 1, the decoherence time for a microtubule is of the order of femtoseconds, 10^{-15} s, or less while typical neuron firing rates are only a few hundred Hz, corresponding to milliseconds.

For our small cat, even near absolute zero, its quantum states are only preserved for a few femtoseconds.

All the foregoing has concerned the superposition of states for a single particle but where multiple particles are in a superposition of states (i.e., entangled) it is possible to separate individual particles so that as long as the individuals are not corrupted by the environment once separated, then decoherence is less of a problem. Thus, if two particles are in a superposition of states so that they are both spin up and down then, when they are separated and one is measured as, say, up, the other measured simultaneously will also be up.

Conclusions

Although this paper originally sought to clarify the position of quantum theory in nanotechnology it has perhaps become more a review of how quantum effects are used in the nanoscale. As research into quantum computing and quantum cryptology continues at pace, it is likely that the boundaries of where quantum effects become important will be pushed further into the larger end of the nanoscale.

It may even become possible, if we can measure time in fractions of an attosecond, to see Schrödinger's cat both alive and dead, although it is also difficult to envisage an assembly of quantum states that would represent the cat dead. Living cells tend to be fairly robust with considerable built-in redundancy and repair capability so the original thought experiment, which effectively amplified the measurement of a superposition of states for a radioactively decaying particle up to the cat's health is not really an example of the cat in both states but merely of its ability to observe quantum effects. However, it is doubtful whether the cat would notice such a rapid change of state and, perhaps, the conclusion is that the interaction of the environment Hamiltonian is such that it favours live cats.

Appendix: some quantum theory

Fundamental to quantum theory is the concept of superposition of states and Heisenberg's uncertainty principle. Quantum theories are defined in terms of an equation for the probability density function, which in the Copenhagen interpretation is the probability of finding the particle or object in a particular state; i.e., its position, momentum, energy at a given epoch and quantum parameters such as spin.

An object (particle, atom or subatomic particle) may be in a superposition of states where, for instance, its spin is both up and down or, for a photon, where spin is realized as helical polarization, having dual polarization. Only when the particle is measured is the quantum state, either spin up or down but not both, realized and the probability density function is assumed to collapse to this measured state. As a consequence it is not possible to exactly know all the parameters of state, as expressed by the Heisenberg uncertainty principle:

$$\Delta p\,\Delta x \geqslant \hbar/2 \text{ and } \Delta E\,\Delta t \geqslant \hbar/2 \qquad \text{(A1)}$$

where p, x, E and t are momentum, position, energy and time, respectively, while $\hbar$ is the reduced Planck constant. This means that for an electron, if its momentum is known exactly then its position is unknown, hence the motion of an electron around the nucleus of an atom tends to be depicted as a cloud surrounding the nucleus.

To investigate the effects of measurement it is necessary for some quantum theory relating to the measurement process. Some basics:

The Hamiltonian or energy operator of the system is given by

$$\mathcal{H} = i\hbar\,\partial_t \qquad \text{(A2)}$$

where ∂_t is the partial derivative with respect to time and i is the square root of -1. If Ψ is the probability density function of the system, then

$$\Psi = C\exp(-i\mathcal{H}t/\hbar) \qquad \text{(A3)}$$

is a solution, where C is a function only of position.

Equations describing the probability density function all take the form

$$U|\varepsilon_0\rangle - \mathcal{H}\Psi = 0 \qquad \text{(A4)}$$

where U is the space propagator and $|\varepsilon_0\rangle$ is the initial state of the particle. For a nonrelativistic object this is given by the Schrödinger equation

$$[-\hbar^2/2m\,\partial^j\partial_j + V]\Psi - \mathcal{H}\Psi = 0 \qquad \text{(A5)}$$

where m is the mass, ∂_j is the 3-space coördinate partial derivative with repeated indices summed from 1 to 3 ($\partial^j\partial_j$ is the Laplace operator) and V is the potential.

The relativistic solution for leptons, the spin-half electron group (also called fermions), is given by the Dirac equation

$$[i\hbar\gamma^j\partial_j - e\gamma^\mu A_\mu + \gamma^0 mc]\Psi - \mathcal{H}\Psi = 0\,, \qquad \text{(A6)}$$

where $\gamma^\mu = (\gamma^j, \gamma^4)$ where γ^4 is the identity and γ^0 are the Dirac matrices formed from the spinor group SU(2) (special unitary complex group of 2 dimensions) and μ are the 4-space indices summed by the metric of 4- dimensional space, e is the electron charge, c is the speed of light, while A_μ are the electromagnetic field potentials.

For the hadrons, protons and neutrons made up of spin-half quarks, the probability density function is given by quantum chromodynamics, where the Dirac equation is modified by the additional gluon field belonging to SU(3).

To consider the effect of measurement it is easiest to envisage a spin-half particle affected by the electromagnetic field. The Stern–Gerlach[12] experiment used magnets to split a particle beam into up and down spin trajectories and then placed a detector in the up spin path. The paths were then recombined. Consequently a particle beam of only up spins would have each particle registered by the detector while a beam of superposition states would have only half the particles registered. The Hamiltonian for the system is given by

$$\mathcal{H} = \mathcal{H}_\mathrm{p} + \mathcal{H}_\mathrm{d} + \mathcal{H}_\mathrm{int} \tag{A7}$$

and

$$\mathcal{H}_\mathrm{int} = \Sigma_a u_{\mathrm{d}a} |{\downarrow}\varepsilon_a\rangle\langle{\downarrow}\varepsilon_a| + \Sigma_a u_{\mathrm{p}a} |{\uparrow}\varepsilon_a\rangle\langle{\uparrow}\varepsilon_a| \,. \tag{A8}$$

$\mathcal{H}_\mathrm{p}$ and $\mathcal{H}_\mathrm{d}$ are the particle and detector Hamiltonians respectively; where $|\varepsilon_a\rangle$ are the orthogonal basis vectors of the energy states of the particle and index $a = 1$ up to the number of such states (for a single electron there is only one state with spin up or down). $u_{\mathrm{p}a}$ and $u_{\mathrm{d}a}$ are the energy interaction coefficients for the particle and detector respectively (our interest is only in the detection interaction with the down spin particle). Its space propagator for the down spin interaction avoiding the detector is represented by

$$U|{\downarrow}\varepsilon_0\rangle = \exp(-i\mathcal{H}t/\hbar)\,\Sigma_a C_a|{\downarrow}\varepsilon_a\rangle = \Sigma_a C_a|{\downarrow}\varepsilon_a\rangle \exp(-iu_{\mathrm{d}a}t/\hbar)\,, \tag{A9}$$

where C_a are the solution coefficients from eqn (A3), and for the up spin

$$U|{\uparrow}\varepsilon_0\rangle = \exp(-i\mathcal{H}t/\hbar)\,\Sigma_a C_a|{\uparrow}\varepsilon_a\rangle = \Sigma_a C_a|{\uparrow}\varepsilon_a\rangle \exp(-iu_{\mathrm{p}a}t/\hbar)\,. \tag{A10}$$

Now suppose that the particle is in a superposition of states

$$|x\rangle = (|{\uparrow}\rangle + |{\downarrow}\rangle)/\sqrt{2} \tag{A11}$$

and the particle has an interaction with the detector so that it propagates according to

$$U|x\rangle|\varepsilon_0\rangle = [\Sigma_a C_a|{\uparrow}\varepsilon_a\rangle \exp(-iu_{\mathrm{p}a}t/\hbar) + \Sigma_a C_a|{\downarrow}\varepsilon_a\rangle \exp(-iu_{\mathrm{d}a}t/\hbar)/\sqrt{2}\,. \tag{A12}$$

If the detector had no effect at all then the probability of finding the particle after detection in a superposition state would be unity. However, the effect of the detector is to change this probability P (no detector) to

$$P(x) = |\langle x|\Psi\rangle|^2 \tag{A13}$$

or

$$P(x) = 1/2\,|\,[\Sigma_b C_b|{\uparrow}\varepsilon_b\rangle \exp(-iu_{\mathrm{p}b}t/\hbar) + \Sigma_b C_b|{\downarrow}\varepsilon_b\rangle \exp(-iu_{\mathrm{d}b}t/\hbar)]\times$$
$$[\Sigma_a C_a|{\uparrow}\varepsilon_a\rangle \exp(-iu_{\mathrm{p}a}t/\hbar) + \Sigma_a C_a|{\downarrow}\varepsilon_a\rangle \exp(-iu_{\mathrm{d}a}t/\hbar)]\,|\,. \qquad \text{A14)}$$

After some calculations using the orthogonality of the basis vectors and the orthogonality of spin, this yields

$$P(x) = 1/4\,\Sigma_b|\,C_b[\exp(-iu_{\mathrm{p}b}t/\hbar) + \exp(-iu_{\mathrm{d}b}t/\hbar)]\,|^2 = 1/2\,\Sigma_b|\,C_b|^2\,\{1 + \cos(u_{\mathrm{p}b}t/\hbar - u_{\mathrm{d}b}t/\hbar)\}\,. \tag{15}$$

[12] This experiment, first carried out in 1922, was used to demonstrate that spin is quantized in elementary particles. What is proposed here is a reversible setup where the superposition particles are recombined.

Where $(u_{pb} - u_{db})\, t/\hbar \ll \pi$, $P(x)$ tends to unity and the superposition state is preserved. This occurs when the detection interaction energy is near zero and thus has no effect on the particle. Consequently the probability of finding the particle in the superposition state $\langle x|$ is 1. The case where $(u_{pb} - u_{db})\, t/\hbar > \pi$ over the detection time will give an average value of ½ for the cosine while $\Sigma_b |C_b|^2$ tends to unity so that the probability tends to a half. Consequently the probability of finding the particle in the superposition state $\langle x|$ is less than 1. Coherence has been lost and the initial quantum state has been destroyed by the measurement.

Decoherence occurs when the interaction energy E is such that

$$E\,t \geqslant h/2\,, \qquad \text{(A16)}$$

which is very similar to the Heisenberg uncertainty principle but now the time is the duration for which the quantum states may be expected to be retained following interference by detection. Note that typical interaction energies represented by the Coulomb potential at the Bohr radius are of the order of a few tens of eV so that the decoherence time is of the order of a fraction of a femtosecond, 10^{-15} s.