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The Cosmos and WIMPs

Graham C. Holt[a]

[a] *Collegium Basilea (Institute of Advanced Study), Basel, Switzerland*

1 Abstract

The last two decades have seen significant technological advances in instruments used to study the universe and subatomic particles: the advent of space telescopes like Hubble and others, gravitational wave detection at Fermilab and ever increasing collision energies at the Large Hadron Collider. These have raised our understanding of the universe but with understanding has come more questions and paradoxes. Both the theory of general relativity and the standard model of quantum theory have continued to be proven, however a proven uniting theory is still elusive. This paper is a review of the theories and in particular one anomaly, that of dark matter, which can only be resolved by the simultaneous application of both theories. The weakly interacting massive article, WIMP, has been a leading candidate for the missing dark matter of the universe but after a decade of searching has not been detected. Recent advances may provide some answers.

2 Introduction

One might wonder what the Cosmos has to do with nanotechnology. The thing about the Cosmos is that distances are huge, the nearest black hole in the star system HR6819, is about 1000 light years away or about 10^{19} metres, while at the nanoscale, the charge radius of the electron is about 10^{-15} metres. The beginning of the universe, the big bang, was energy in the form of bosons, the force carrying subatomic particles of electromagnetism, radiation or the weak and strong nuclear forces and gravity (although a boson for gravity has not been detected). The space containing this energy expanded rapidly at the speed of light in the initial inflation period and subatomic particles such as electrons, protons and neutrons were formed after about 1 μs. A further 300 thousand or so years later, the beginning of the visible universe, the sub atomic particles had cooled sufficiently that hydrogen atoms were formed while photons could escape without collision. These subatomic particles which are at the smallest end of the nano scale continue to play an important part in the Cosmos but the two theories, one for the large-scale given by Einstein's General theory of relativity and the other, on the small scale, known as Standard Quantum theory, have never been easy bedfellows.

General relativity has been particularly successful in explaining planetary motion enabling probes to be sent to distant planets, gravitational lensing, black holes (recently visualised by radio telescopes working together[1]) and more recently gravitational waves. In the 1970s, Hawking applied some quantum theory to black holes and demonstrated their evaporation; not

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[1] First ever real image of a black hole revealed, New Scientist,April 2019, https://www.newscientist.com/article/2199330-first-ever-real-image-of-a-black-hole-revealed/

so far observed but which has created another paradox concerning loss of information when matter falls into the black hole. In quantum chromo-dynamics, recent discoveries with the Large Hadron Collider have demonstrated the predictions of the Standard theory with the observation of leptons, the electron family, and hadrons, the quark family, and now the Higgs boson which provides a mechanism by which electrons and quarks are formed from pure energy.

In order to describe the Cosmos as a static or slowly expanding universe, Einstein had to add a cosmological constant to the very elegant tensor equations, eq.1. The solution represents the universe as we now see it, which is flat everywhere except close to massive objects such as stars or black holes, and such that it expands and expands forever or collapses to then re-expand. It is a gloomy future if the universe continues to expand at an increasing rate for the assumption is that there is a fixed mass inclusive of energy, wherein with masses moving apart and the force of gravity diminished, then new stars are not manufactured. Thus when all the suns have become white dwarfs, neutron stars or black holes the cosmos will become cold and dark, and very lonely. Einstein was not happy with the constant nor with quantum theory where he famously quoted, " God does not play dice". The problem is that the cosmological constant may be removed if there exists negative pressure to slow the expansion and contraction of the universe given by the solution with just an initial mass. More recently, another problem has arisen in that the outer stars of spiral galaxies are observed as rotating around a central massive black hole too fast[2]. This requires extra nebulous matter permeating the universe but gathered by the gravitational pull of large galaxies into a halo of invisible

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[2] A simple calculation shows that adding effective, albeit in a halo, central mass increases the rotation speed at a given distance from the central black hole

dark matter. This matter can have no interaction with other matter other than through gravity and must be thinly dispersed so as not to impede the trajectory of photons. These are the so-called weakly interacting massive particles, WIMPs.

So far the Standard theory of quarks, quantum chromo-dynamics has no place for WIMPs and attempts to find such particles with the Large Hadron Collider have failed, implying that their mass must be in the region of TeV (tera 10^{12} eV) or about the mass of 1000 times the mass of a hydrogen atom. Since the planets in the solar system show no perturbation to such a nebulous mass it puts limits on the density. The universe has current observed average density of about 6 protons per cubic metre and dark matter in the milky-way galaxy halo is thought to be about 100 times more[3]. Note, interstellar density is about ten orders of magnitude less than the average.

3 Problems with gravity

There are at least three problems with Einstein's general relativity. The first is the negative pressure field of dark energy, now regarded as a negative mass repulsing gravity but still resulting in a greater expansion rate than proposed by Hubble[4]. There is currently no explanation for this field only that the mass of this energy must make up a large percentage, 68 %, of the mass of the universe in order to have a stably expanding solution which requires the cosmological constant, c.f eq.2. This all pervading invisible force only has affect on the large-scale and only inter-

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[3] Dark Matter in the Solar System, X. Xu and E. R. Siege, June 2008, https://arxiv.org/pdf/0806.3767.pdf

[4] Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics Beyond LambdaCDM," Adam G. Riess et al., 2019, Astrophysical Journal, arxiv.org/abs/1903.07603

acts through mass.

The second problem concerns evaporating black holes and the loss of quantum information as material falls in towards the singularity. The baryon number, the number of hadrons namely quarks, in the universe should be conserved. Hence the problem when matter falls through the event horizon of a black hole and is effectively lost to the rest of the universe. The Sun, if it collapses into a black hole, will have reduced its radius from 7 10^8 metres to the event horizon at 2.9 kilometres (the sun's mass converted to distance by the gravitational constant is 1.4 10^3 metres). Postulates to overcome this paradox include eventual evaporation releasing the hadrons or conservation over multiple universes where the matter inside a black hole reaches the singularity and explodes in a white hole in another universe.

The third problem, explored here, should be less intractable. The 27 % of the mass of the universe in dark matter must be clustered round the large objects of the universe moved by the action of gravity. Moreover, from many body analysis, the dark matter must form a halo around the various masses for otherwise, the dark matter near planets and stars would be drawn to them and coalesce. This means that after the big bang, if dark matter was homogeneously distributed, then by now the Earth should contain some. Its detection will be difficult as it is so sparsely distributed by comparison to the solid objects comprising the planet. However, recent advances, like the detection of gravitational waves and the next round of increasing the energy of collisions in the LHC gives grounds for hope.

The Laser Interferometer Gravitational-Wave Observatory, LIGO, measuring the distance to a reflecting mirror some four kilometres away in two arms at right angles is able to detect the strain in the Earth; equivalent to measuring the distance change of about the width of a proton. The effect of the wave is to squeeze and then expand the Earth as it passes through. So far, relatively small back holes rotating about each other and eventually colliding to form a single black hole of combined mass have been detected from the gravitational waves produced. The gravitational waves are distinctive, characteristic of the black hole sizes and distance from each other while the intensity of the wave decreases with distance rather than distance squared as for electromagnetic radiation[5]. With two LIGO detectors, the direction is apparent and has enabled corroboration with radiation (matter falling into the black hole from its accretion disc which radiates as it is accelerated) from known black holes. More easily aligned with visible observations are coalescing neutron stars. It is hoped with more detectors that gravitational waves will provide more detail of the early universe and possibly the formation of dark matter. Gravitational waves have frequency in the audible range with the smallest black holes and neutron stars giving the highest frequency, which implies that dark matter if having formed small primordial black holes when it was dense in the early universe, would probably produce very high frequency waves. Moreover, if sufficient primordial black holes are found in the region before the visible universe was formed it may obviate the need for dark energy.

4 Problems with quantum theory

The other hopeful advance is with the LHC operating at ever higher energies. However, most of the interactions and particles predicted within the Standard theory which includes quan-

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[5] Observation of Gravitational Waves from a Binary Black Hole Merger, B.P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration),Phys. Rev. Lett. 116, 061102 Published 11 February 2016

tum chromo-dynamics, quantum electrodynamics and weak nuclear decay have now been observed. Quantum electrodynamics started with the very elegant Dirac equation, eq.3 for the electron family and then developed into a scattering theory, quantum electrodynamics, eq.4, by Feynman. It was observed that the probability density function is invariant under phase transitions, corresponding to phase invariance of the electromagnetic four dimensional vector potential carried by the photon. This phase invariance or gauge theory[6] gave rise to quantum chromodynamics in the late 1960s where the phase invariance applies to the eight four dimensional gluon potentials, eq.5. The next stage was to include invariance of the three weak force W+, W- and Z bosons and finally the inclusion of a scalar field representing the Higgs boson. Notice that the groups involved are U(1) for the photon, SU(3) for the quark gluons and SU(2) for the W,Z bosons(weak nuclear decay) where generators of the groups provide the interaction potentials[7]. The difficulty with these groups is that they function properly only in flat space. Hence, the difficult marriage of general relativity with quantum theory. One possible solution is string theory, where particles are represented by vibrating strings at the Planck length[8] and the group functions are imbedded by an extra seven dimensions of flat space. So far, there is no physical evidence that this theory is correct.

Another possibility, called supersymmetry takes a more general gauge theory of the four dimensional Poincaré subgroup which gives rise to massive shadow particles. The simplest gauge group unifying the SU(3), SU(2), U(1) groups is SU(5) or SO(10),

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[6] The gauge is a means of specifying the extra degrees of freedom of the Lagrangian

[7] U(1) is the unitary group of 1 complex dimension while SU(N) is the special unitary group of N complex dimensions.

[8] The force of gravity between photons separated by this length is of the same order of the other forces.

requiring at least 6 extra dimensions. However, the lightest of such shadow particles ought to have been detected by the LHC. The advantage of this theory is that there are WIMP candidates requiring a yet undetected new boson field.

5 Some elegant mathematics

5.1 General Relativity

Einstein's equation for general relativity is

$$\mathrm{R}_{\mu\nu} - 1/2 g_{\mu\nu} R = 8\pi G/c^4 T_{\mu\nu}, \tag{1}$$

where the Ricci tensor $R_{\mu\nu} = g^{\alpha\beta} R_{\mu\alpha\nu\beta}$; the Reimann tensor, $R_{\mu\alpha\nu\beta}$, of the second derivative of the metric, $g_{\mu\nu}$, defines space time curvature, while $T_{\mu\nu}$ is the stress energy tensor and G the gravitational constant. The 4-space time indices are μ,ν etc. For most exact solutions for the metric, the tensor $T_{\mu\nu}$ is zero. For gravitational waves the mass density term represents the two rotating masses[9].

For expanding, mostly flat space, the stress energy tensor has a mass density and includes the extra cosmological constant term

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[9] Gravitational Waves: An Introduction, Indrajit Chakrabarty, 21 Aug 1999 , https://arxiv.org/pdf/physics/9908041.pdf

$-\lambda g_{\mu\nu}$ which represents an initial mass and a repulsive force[10];

$$\mathrm{R}_{\mu\nu} - 1/2 g_{\mu\nu} R - \lambda g_{\mu\nu} = 8\pi G/c^4 T_{\mu\nu}.$$
(2)

5.2 Standard Quantum theory

The Dirac Lagrangian for leptons, the spin half electron group of fermions, is given by

$$\mathbf{L}_e = \int dx^4 \bar{\Psi}[\gamma^\mu(i\bar{h}\partial_\mu - eA_\mu) - mc]\Psi,$$
(3)

where γ^μ are the Dirac matrices formed from the spinor group SU(2) and μ are the 4-space indices summed by the metric, while A_μ are the electromagnetic field potentials and Ψ is the probability density function of the lepton of mass m and charge e. $\bar{h}$ is Planck's constant divided by 2π and c is the speed of light.

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[10] COSMOLOGICAL MODELS,GEORGE F R ELLIS and HENK VAN ELST, NATOAdv.StudyInst.Ser.C.Math.Phys.Sci.541:1-116,1999, https://arxiv.org/pdf/gr-qc/9812046.pdf

The full quantum electrodynamic Lagrangian is given by

$$\mathbf{L}_{QED} = \int dx^4 \bar{\Psi}[\gamma^{\mu}(i\bar{h}\partial_{\mu} - eA_{\mu}) - mc]\Psi - 1/4F_{\mu\nu}F^{\mu\nu}, \quad (4)$$

which also generates Maxwell's equations. The covariant derivative $(i\bar{h}\partial_{\mu} - eA_{\mu})$ and probability function Ψ are invariant under the gauge transformation $\Psi \to e^{i\theta}\Psi$. The tensor $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$. The gauge group of phase θ is U(1).

The chromo-dynamics gauge group is SU(3) where the transform is $\Psi \to e^{i\lambda_a \chi^a}\Psi$ and the λ_a are the 8 matrix generators of the group and χ^a is a vector phase. The quark Lagrangian is then

$$\mathbf{L}_{quark} = \int dx^4 \bar{\Psi}_q[\gamma^{\mu}(i\bar{h}\partial_{\mu} - g\lambda_a A^a_{\mu}) - m_q c]\Psi_q - 1/4G^a_{\mu\nu}G^{\mu\nu}_a, \quad (5)$$

where m_q is the quark mass and the A^a_{μ} are the 8 gluons.The probability density function Ψ_q with its 3 complex SU(3)vector components represents the 3 quark colours to form a nucleus. The coupling constant g is also then part of the gauge invariant gluon equations generated by $G_{\mu\nu a} = \partial_{\mu}A_{\nu a} - \partial_{\nu}A_{\mu a} + gg_{abc}A^b_{\mu}A^c_{\nu}$. The antisymmetric term g_{abc} is the group structure constant for SU(3).

A similar Lagrangian is constructed for the W,Z bosons, W^i from the gauge invariance of the SU(2) generators, the 3 Pauli matrices σ_i.

Finally the Higgs boson field is scalar but with quadratic probability density function [11].

6 Where next

Is there room for a more complex Higgs boson? The Higgs is a scalar boson with a quartic potential of energy versus probability density and taking the form of a Mexican hat, see fig.1. The consequence is that the Higgs boson has rest mass which it can lose when interacting with another particle leaving the boson with minimum energy (the rim of the hat and highest probability). Only the photon, electron, neutrinos and particular combinations of three quarks (proton and neutron in the nucleus) are stable, for instance the Higgs has a life of 10^{-22} s. Yet the WIMP must be stable and so should be more easily detectable. It is conceivable that the Earth's fossil record could contain tiny amounts of this locally very dense material unlike other matter with no orbiting electrons.

The next round of LHC collisions at TeV energies - the Higgs boson is about 125 GeV[12] - could produce some new particles of which if the WIMP was one, it would be characterised by always falling down and carrying away some of the momentum of collision. However, a huge underground tank of xenon has so far failed to detect any WIMPs falling into the Earth. The

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[11] THE STANDARD MODEL OF ELECTROWEAK INTERACTIONS, A.Pich, CERN-2012-001, pp.1-50, January 2012, https://arxiv.org/abs/1201.0537

[12] Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Physics Letters B, Volume 716, Issue 1, 17 September 2012, Pages 30-61

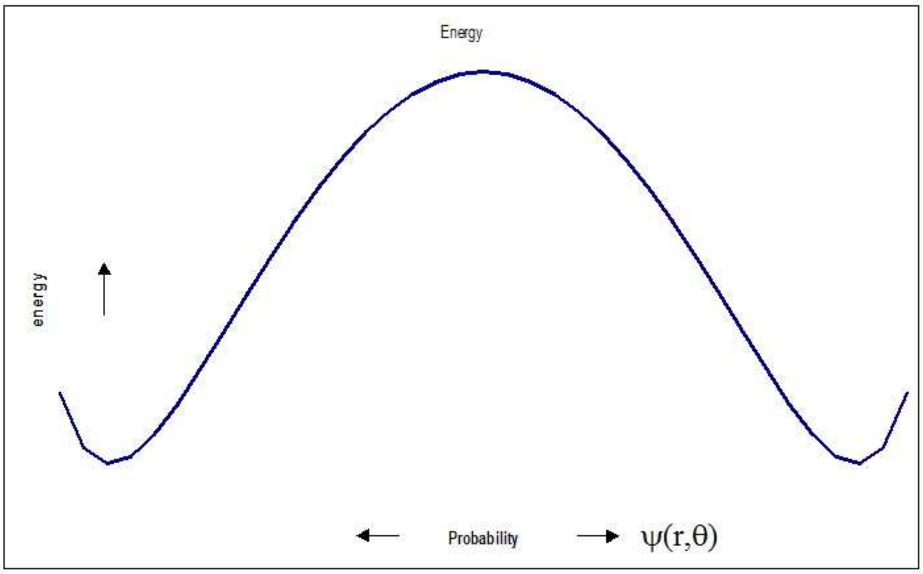

Figure 1: Mexican Hat Potential

detector relies on a collision of a WIMP with a xenon molecule causing the release of a photon when the collision momentum is absorbed but so far there is no evidence of the elusive particle[13].

Unfortunately after nearly five years there is still still no detection of a WIMP and other candidates, less massive, are now being studied such as neutrinos with mass less than a millionth that of an electron. Neutrinos interact weakly through the W and Z bosons but are mostly not detected but if in sufficient abundance orbiting super massive black holes could account for some of the dark matter[14].

Some observations of galactic explosions near a black hole emitting gamma rays have shown that over billions of light years of

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[13] First Dark Matter Search Results from the XENON1T Experiment,E. Aprile et al. (XENON Collaboration),Phys. Rev. Lett. 119, 181301 Published 30 October 2017

[14] DARK MATTER AND NEUTRINOS,G. Sharma, A and B. C. Chauhan, https://arxiv.org/pdf/1711.10564.pdf, Nov 2017

travel, that possibly light speed is not constant[15]. High energy photons of very short wavelength appear to reach Earth slower than lower energy less massive photons. Is this a photon graviton interaction at the level of the space-time quantum foam [16] at the Planck length ($\sqrt{\hbar G/c^3} = 10^{-35}$ metres[17]) implying that the speed of light is not constant, requiring new physical laws and modifying both General Relativity and the Standard model? If the speed of light is proved not to be constant then it is likely that the describing equations will not be elegant.

There is therefore a dilemma; could Einstein's general relativity require extra terms to describe the scale of the cosmos, perhaps a time/scale dependent gravitational constant obviating the need for WIMPS, or is there more to the Standard theory of the quantum world?

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[15] Albert J, Ellis J, Mavromatos NE, Nanopoulos DV, Sakharov AS, Sarkisyan EKG (2008). "Probing quantum gravity using photons from a flare of the active galactic nucleus Markarian 501 observed by the MAGIC telescope". Physics Letters B. 668 (4): 253257

[16] The fluctuation of space time on very small scales due to quantum mechanics

[17] The identity arises from the Heisenberg uncertainty principle for the smallest measurable length between two photons acting under gravity. It assumes that two photons of mass derived from their wavelength when attracted by gravity have momentum and wavelenghth obeying Heisenberg's uncertainty principle