Negative index of refraction and metamaterials

Graham C. Holt

Collegium Basilea (Institute of Advanced Study)

Introduction

Maxwell's equations describing the electromagnetic field have been known and studied for over a century. It is perhaps a reflexion of the growing confidence of scientists employing mathematics that theoretical prediction is now eagerly awaited, and often soon demonstrated as reality. The positron predicted in 1920 by Dirac was initially regarded as an artifact of the mathematics, and Dirac was reticent about publishing the consequences of his prediction as the existence of a new particle. In a less topical area of theoretical physics, overshadowed for many years by fundamental particle research, it is therefore perhaps not surprising that Veselago, a Russian scientist who predicted negative refraction in 1966,[1] was largely forgotten until the 1990s. Now with the demonstration in 2000 of a constructed material showing negative refraction at microwave frequencies,[2] further theoretical predictions are eagerly awaited for their subsequent realization in practical materials. Interest in them and their applications has grown exponentially, and in particular the search to find a constructed or metamaterial that operates in the visible region of the spectrum. Applications have been postulated ranging from cloaking devices and superlenses to optical switches and elements of a photon computer.

Theory

The wave solution to Maxwell's equations at an interface gives rise to Snell's law of refraction, derived by assuming continuity of the wave function across the interface:

$$n_1\sin(\theta_1) = n_2\sin(\theta_2)\,. \tag{1}$$

The refractive index is defined as

$$n = (\varepsilon\mu)^{1/2} \tag{2}$$

where ε is the permittivity and μ is the magnetic permittivity. Until Veselago, n was always taken as the positive square root, but if ε and μ are both negative then the negative value of n is appropriate, giving rise to the bending of the light as in a convex lens, see Figure 1.

---

[1] V.G. Veselago, The electrodynamics of substances with simultaneously negative values of ε and μ. *Soviet Phys. Usp.* **10** (1968) 509–514 (first published in Russian in 1966).

[2] D.R. Smith, W.J. Padilla, D.C. Vier, S.C. Nemat-Nasser and S. Schultz, Composite medium with simultaneously negative permeability and permittivity. *Phys. Rev. Lett.* **84** (2000) 4184–4187.

Nanotechnology Perceptions **4** (2008) 201–205
Received 14 July 2008

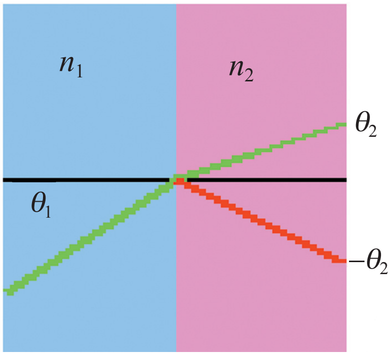

Figure 1. Snell's law at the interface of two materials. Green is the normally refracted ray while the red ray results from one material having negative refractive index.

Maxwell's equations are:

$$\nabla\wedge\mathbf{E} = -c^{-1}\partial_t\mathbf{B} \tag{3}$$

$$\nabla\wedge\mathbf{H} = c^{-1}\partial_t\mathbf{D} + 4\pi\mathbf{J} \tag{4}$$

$$\mathbf{D} = \varepsilon\mathbf{E} \tag{5}$$

$$\mathbf{B} = \mu\mathbf{H} \tag{6}$$

where **E** and **H** the electric and magnetic fields, and **D** and **B** are the dielectric displacement and magnetic inductive fields, respectively. The current density is **J**. The wave solution for the equations is given by

$$\mathbf{E} = \mathbf{K}\wedge\mathbf{H}_0 e^{i(\mathbf{K}\cdot\mathbf{x} - \omega t)} \tag{7}$$

$$\mathbf{H} = \mathbf{K}\wedge\mathbf{E}_0 e^{i(\mathbf{K}\cdot\mathbf{x} - \omega t)} \tag{8}$$

where **K** is the phase propagation direction and ω the angular frequency, provided

$$\mathbf{K} = \frac{c\omega}{n}\hat{K}, \tag{9}$$

and $\hat{K}$ is the unit vector. Note that the energy flux or Poynting vector is given by

$$\mathbf{P} = \mathbf{E}\wedge\mathbf{H} \propto \mathbf{K}. \tag{10}$$

Thus **E**, **H** and **K** a form a right-handed vector space provided n is greater than zero. If it is less than zero then we have a left-handed vector space, which has given rise to the term left-handed materials exhibiting negative refractive index. Negative refractive index also then gives backward Cerenkov radiation and reversed Doppler shift.

Assuming the permittivities are functions only of position in the form

$$\varepsilon = \varepsilon_0 e^{i\mathbf{a}\cdot\mathbf{x}} \tag{11}$$

and

$$\mu = \mu_0 e^{i\mathbf{b}\cdot\mathbf{x}}, \tag{12}$$

then the complex permittivities will give rise to currents in the material if it is a conductor, and by solving Maxwell's equations, the permittivity vectors will be functions of angular frequency ω. For nonconducting materials, solutions still exist and the permittivity vectors indicate regions of inhomogeneity in the material.

By solving Maxwell's equations for a matrix material, Pendry was able to calculate the allowable fields and the transmission of the medium.[3] The medium was a material filled with cavities and the overall effect in a layered photonic crystal consisting of layers of positive and negative refraction at microwave frequencies gives a lens.[4]

The underlying mechanism of permittivity depends on the energy or wavelength of the incident photons. In translucent material some of the photon energy is absorbed while most is transmitted at a refracted angle given by Snell's law.

For long wavelength radiation (near microwave frequencies), the incident radiation creates atomic dipole moments, which heat the material and reduce the phase velocity.

At higher frequencies, microwave to ultraviolet, wavelength to about 100 nm, electrons in the atoms and molecules of the material absorb energy by being raised to higher energy bands (as in, for example, the microwave heating of water).

At very high frequencies, far ultraviolet to X-rays, wavelength 10^{-3} nm, the atomic structure acts as a diffraction grating and Snell's law is inappropriate.

The first negative refractive index materials were constructed in 2000 using micromachining techniques to control the electromagnetic field at microwave frequencies (centimetre wavelength). The negative magnetic permittivity was created using split ring resonators, see Figure 2 (left), where the effective printed circuit generates circulating currents giving a permittivity

$$\mu = 1 + \frac{F\omega^2}{\omega_0{}^2 - \omega^2 - i\omega\mathbf{\Gamma}} \tag{13}$$

where F and $\mathbf{\Gamma}$ depend on the geometry and ω_0 is the resonant frequency of the equivalent LC circuit. Then, $\omega > \omega_0$ results in negative permittivity. Similarly the electric permittivity using an array of conducting wires, Figure 2 (right), is given by

$$\varepsilon = 1 + \frac{\omega_p{}^2}{\omega_0{}^2 - \omega^2 - i\omega\mathbf{\Gamma}} \tag{14}$$

where the metal plasma frequency at which the metal becomes transparent is

$$\omega_p{}^2 = 1 + \frac{4\pi n e^2}{m^*} \tag{15}$$

where n is the carrier density and m^* the effective mass of the charge carriers. The plasma frequency results from the displacement of the conduction electrons in the metal by the electric field created by the incident photons. At photon frequencies greater than this, the electrons present no barrier to the incident radiation.

Putting the two structures (Figure 2) together to make an array then gives a negative refractive index material, see Figure 3. In terms of using negative refraction to construct a flat lens, one concern is the diffraction limit of the resolved image, which is typically $\lambda/2$. However,

---

[3] G. Guida, P.N. Stavrinou, G. Parry and J.B. Pendry, Time-reversal symmetry, microcavities and photonic crystals. *J. Modern Optics* **48** (2001) 581–595.

[4] I.V. Shadrivov, A.A. Sukhorukov and Yu.S. Kivshar, Beam shaping by a periodic structure with negative refraction. *Appl. Phys. Lett.* **82** (2003) 3820–3822.

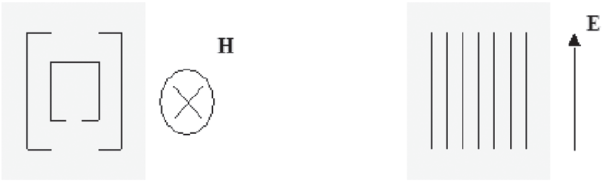

Figure 2. Circuit to induce circulating current from the magnetic field, left (perpendicular to the page), and from the electric field, right (parallel to the page). For the magnetic field, the almost circular conducting tracks are all capacitatively linked, forming a classic *LC* resonant circuit with the magnetic field generating current in the loops. The electric field generates current in the wires, which are also capacitatively linked to form a transmission line with characteristic resonance.

unlike conventional lenses, which lose the evanescent field, negative refractive index materials are able to capture this energy,[5] and in 2004 a resolution of λ/5 was demonstrated.[6] A problem with constructed metamaterials is that the conducting element will become transparent at the plasma frequency, which for most metals is near the ultraviolet. The theoretical demonstration that photonic materials with internal cavities may be used to create negative refractive index has provided further impetus to develop crystals operating in the visible region.

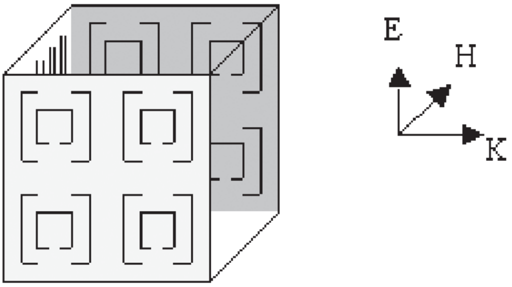

Figure 3. The electric and magnetic "circuits" are printed on either side of a circuit board and assembled to form an array or lattice, in which each side of the box structure forms one side of the next box. The field directions and phase propagation direction are shown for the left-handed composite.

Current status

The first confirmed demonstration of negative refractive index was by Shelby et al. in 2001,[7] using the structure of Figure 3. The structure with the two circuits printed on either side of the printed circuit board confirmed a negative refractive index; each circuit was about a third of a centimetre square. Since then, the principle has been extended to higher frequencies by building similar circuits on silicon substrates with the individual circuits being of the order of 100 nm, and negative refractive index has been demonstrated in the near infrared. Moreover, since the negative permittivity is a function of an effective *LC* circuit and this in turn tends to give fairly narrow resonances, the spectrum transmitted with negative refractive index tends to be both narrow and tunable. This has led to the concept of controlling the *LC* network with an integral

---

[5] J.B. Pendry, Negative refraction makes a perfect lens. *Phys. Rev. Lett.* **85** (2000) 3966–3969.

[6] A. Grbic and G.V. Eleftheriades, Overcoming the diffraction limit with a planar left-handed transmission-line lens. *Phys. Rev. Lett.* **92** (2004) 117403.

[7] R.A. Shelby, D.R. Smith and S.Schultz, Experimental verification of a negative index of refraction. *Science* **292** (2001) 77–79.

semiconductor. Such dynamic tuning was first demonstrated by Padilla,[8] and has raised the spectre of fast photon switches. Here the switching would be at electron conduction speeds but the data through the logic gates would be travelling at light speed.

The search for the 'Holy Graal' of negative refractive index in the visible spectrum has led to studies of doped crystals with surprising results. Magnetodielectric particles embedded in the crystal matrix[9] give negative refractive index even though the real parts of the permittivities are positive. Superlattices at millimetre wavelength, utilizing a multilayer stack of ferromagnetic and superconducting thin films of $Y\,Ba_2Cu_3O_7$ layers to provide negative permittivity while negative magnetic permittivity is achieved via ferromagnetic $(La{:}Sr)MnO_3$ layers,[10] are also showing promise in the development of superlenses. More recently, near-ultraviolet negative refractive index has been demonstrated in an ultrathin $Au–Si_3N_4–Ag$ waveguide.[11]

Although photonic crystals with internal cavities were originally studied for their forbidden frequency or band gap properties as light filters, there is now interest in their property of nonlinear negative refractive index.[12] These lithium niobate crystals doped with iron give nagative refractive index in the infrared.

The science fiction "must have" for any self-respecting space cruiser let loose in the galaxy is a cloaking device. The advent of negative refractive index makes this a possibility, albeit at present only at quite specific frequencies of the visible spectrum, by bending light round the object to be concealed. A copper cylinder was made invisible in the path of a radar beam by surrounding it in composite metamaterials in 2006.[13] The surrounding metamaterial was centimetres thick, and the question now arises as to whether the technique can be scaled to visible frequencies, especially since the new breed of metamaterials operating at visible frequencies tend to have high transmission loss, hence only a few percent of the incident light intensity is transmitted. A recent report suggests that in theory a multilayered metamaterial sphere could cloak a 200 nm diameter object in its centre over a 100 nm spectral width in the ultraviolet.[14]

Although cloaking devices have captured the imagination of the more popular science press there would seem to be a lot more technology required than is presently available to cloak a large object over the whole visible or radar spectrum. In all likelihood the less eye-catching application as elements of a photon computer will make most progress in the immediate future.

---

[8] W.J. Padilla, A.J. Taylor et al., Dynamical electric and magnetic metamaterial response at terahertz frequencies. *Phys. Rev. Lett.* **96** (2006) 107401.

[9] Y.-F. Chen, P. Fischer and F.W. Wise, Negative refraction at optical frequencies in nonmagnetic two-component molecular media. *Phys. Rev. Lett.* **95** (2005) 067402.

[10] A. Pimenov, A. Loidl, P. Przyslupski and B. Dabrowski, Negative refraction in ferromagnet-superconductor superlattices. *Phys. Rev. Lett.* **95** (2005) 247009.

[11] H.J. Lezec, J.A. Dionne and H.A. Atwater, Negative refraction at visible frequencies. *Science* **316** (2007) 430–432.

[12] A. Dreischuh, D.N. Neshev, V.Z. Kolev, S. Saltiel, M. Samoc, W. Krolikowski and Yu.S. Kivshar, Nonlinear dynamics of two-color optical vortices in lithium niobate crystals. *Opt. Express* **16** (2008) 5406–5420.

[13] D. Schurig, J.J. Mock, B.J. Justice, S.A. Cummer, J.B. Pendry, A.F. Starr and D.R. Smith, Metamaterial electromagnetic cloak at microwave frequencies. *Science* **314** (2006) 977–980.

[14] J. Elser and V.A. Podolskiy, Scattering-free plasmonic optics with anisotropic metamaterials. *Phys. Rev. Lett.* **100** (2008) 066402.